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Influence of Particles on Radiative Base Heating from the Rocket Exhaust Plume

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The modified discrete-ordinates method with a two-phase mixture of nongray gases and a cloud of particles is applied to investigate the radiative base heating due to plume and searchlight emission. To obtain the radiative heat flux on the base plane, the exhaust plume is considered as an absorbing, emitting, and scattering medium, while the environment between plume boundary and rocket base plane is assumed nonparticipating. The radiative properties involving nongray gas and particle behavior are modeled by using the weighted sum of gray gases model with particles. After validating the present two-phase radiation solutions, numerical investigations are made to examine the effects of such various parameters as particle concentration, scattering phase function, particle temperature, and nongray gas composition on the radiative base heating in an isothermal cylindrical plume with nongray gases and Al_2O_3 solid particles. The radiative base heating increases as the particle concentration and temperature increase. The forward scattering of particles increases plume emission while it decreases searchlight emission. The gas composition, however, does not significantly influence the radiative base heating.

Nomenclature

A_j	=	expansion coefficient of the scattering phase function, Eq. (11)
a_I	=	coefficient of the discretization equation at nodal point I, Eqs. (6) and (7)
C_p	=	particle concentration, kg/m^3
D_i^{mn}	=	directional weights in direction mn at surface i, Eq. (8)
D_r^{mn}	=	directional weights in r direction
$E_{b,\text{ref}}$	=	blackbody emissive power at T_{ref}, $=\sigma T_{\text{ref}}^4$, W/m^2
I_b	=	blackbody radiative intensity, $=\sigma T^4/\pi$, W/(m$^2\cdot$sr)
K	=	number of total gray gases, Eq. (1)
M, N	=	total number of radiation direction in θ and ϕ directions
N_i	=	particle number density pertaining to group i
N_r, N_z, N_θ, N_ϕ	=	number of grids in r, z, θ, and ϕ directions, respectively
$\boldsymbol{n}_i$	=	outward unit normal vector at surface i
q	=	radiative heat flux, W/m^2, Eq. (1)
R_{ex}	=	radius of nozzle exit
r_{pi}	=	particle radius pertaining to group i
T	=	temperature, K
w	=	weighting factor for nongray gases model
Z_{pl}	=	plume length
ΔA_i, ΔV	=	surface area and volume of the control volume, respectively
$\Delta\Omega^{mn}$	=	discrete control angle, sr
ε_p	=	particle emissivity
θ	=	polar angle measured from the z axis, rad
κ, σ_s	=	absorption and scattering coefficients, respectively, m^{-1}
μ, η, ξ	=	direction cosines in the r, ϕ, and z directions, respectively, that is, $\boldsymbol{s}=\mu\boldsymbol{e}_r+\eta\boldsymbol{e}_\phi+\xi\boldsymbol{e}_z$
Φ	=	scattering phase function, sr^{-1}
ϕ	=	azimuthal angle measured from the x axis, rad
ω	=	weighting factor

Subscripts

E, W, T, B	=	East, West, top, and bottom neighbors of P
e, w, t, b	=	East, West, top, and bottom control volume faces
g	=	gas
k	=	kth gray band
P	=	nodal point in which intensities are located
p	=	particle
μ, η, ξ	=	direction cosine

Superscripts

m, n	=	radiation direction in θ and ϕ directions, respectively

Introduction

DEVELOPMENT of new propellants containing energetic binders and additives such as boron and aluminum greatly increases the importance of radiative base heating from rocket exhaust plumes via searchlight and/or plume emission. Intrinsically

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the properties of gases and particles within exhaust plumes influence the thermal radiation process through absorbing, emitting, and scattering characteristics. Therefore, models as well as methods for predicting rocket plume base heating are in high demand.

Numerous practical engineering methods and models for calculation of the radiative base heating have been developed and widely used. While Tien and Abu-Romia [1] considered the rocket exhaust plume as a semi-infinite cylindrical absorbing and emitting gas body with uniform temperature and properties, Morizumi and Carpenter [2] developed a technique calculating rocket base and spacecraft heating environments due to particle radiation from the exhaust plume of an aluminized composite propellant rocket by treating radiation from a cloud of particles as that from an equivalent radiative surface. The thermal radiation from a cylindrical cloud of absorbing, emitting, and anisotropically scattering particles has been investigated by Stockham and Love [3] using the Monte Carlo method. It was found that the anisotropic scattering and searchlight emission play an important role in radiative base heating. Watson and Lee [4] employed the Monte Carlo method to model the solid rocket booster plumes and considered axial and radial variations of the plume properties. Nelson [5] investigated the influence of radiation scattering on the infrared radiation signature of representative plumes from four types of tactical rocket using the SIRRM numerical code. Everson and Nelson [6] adopted the backward Monte Carlo method which is more computationally efficient than the direct Monte Carlo method to predict radiative heating from emission and scattering by both nongray gas and particles. Recently, Baek and Kim [7] and Tan et al. [8] investigated the base heating due to searchlight and/or plume emission by using the finite volume and backward Monte Carlo methods, respectively.

The objective of this work is to investigate the effects of two phase mixtures of nongray gases with particles on the radiative base heating due to searchlight and plume emission. The exhaust plume is considered an absorbing, emitting, and isotropically or anisotropically scattering medium, while the environment between the plume boundary and the rocket base plane is assumed nonparticipating. The gases absorb and reemit but do not scatter radiation, whereas particles scatter radiation but absorb and reemit only for large particles at higher temperatures. The radiative properties involving nongray gas and particle behavior are modeled by using the weighted sum of the gray gases model (WSGGM) with particles. The scattering phase function is approximated as a finite series of Legendre polynomials. After validating the present two-phase radiation solutions by comparison with those of previous works, a detailed investigation of the radiative base heating due to searchlight and/or plume emission is conducted and examined by changing various parameters such as particle concentration, scattering phase function, temperature, and nongray gas composition.

Mathematical Formulation

Models for Rocket Plume Base Heating

A schematic of the radiative base heating due to "searchlight emission" and "plume emission" is shown in Fig. 1, where δ, R_{ex}, and Z_{pl} are the plume cone angle, the radius of the nozzle exit, and the plume length, respectively. The exhaust plume emerging from the nozzle exit at $z = 0$ has a cylindrical shape when $\delta = 0$ deg and a finite conical shape when $\delta \neq 0$ deg. Searchlight emission is caused by photons which are emitted from the inside of the rocket nozzle and then scattered by the exhaust plume toward the base plane, while plume emission is due to photons emitted directly from the exhaust plume as shown in Fig. 1.

The isothermal cylindrical exhaust plume is treated as a radiatively participating two-phase mixture of nongray H_2O, CO_2, and other transparent gases with a cloud of particles. The particles are assumed to be spherical and to have five different diameters. This plume is surrounded by a cold and nonparticipating environment. No other external incidence of radiant energy is considered here and the plume boundary is considered free. For simplicity, the exhaust plume is assumed to be uniformly distributed with a gas temperature of T_g and a particle temperature of T_p, and the base plane is cold and black.

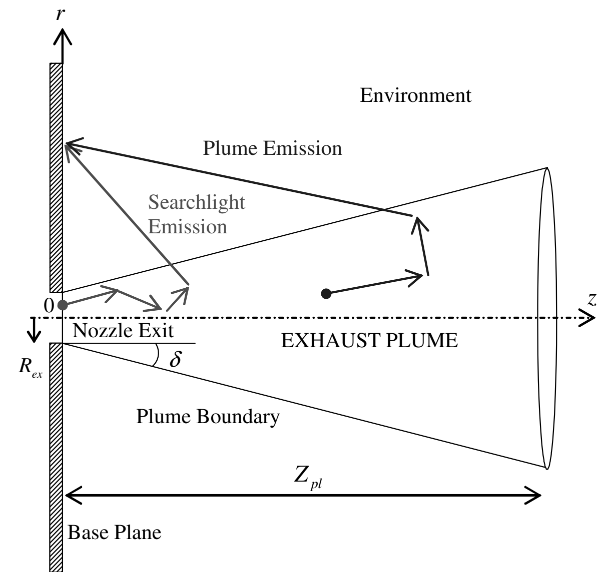

Fig. 1 Schematic of the rocket plume base heating due to searchlight and/or plume emission.

Radiative Transfer Equation

For a nongray mixture of gases, the radiative heat flux measured on the base plane is defined as the summation of the kth gray band intensity such as

$$q_z = \sum_k^K \int_{\Omega=4\pi} I_k(\boldsymbol{r}_w, \boldsymbol{s})(\boldsymbol{s} \cdot \boldsymbol{n}_w)\, \mathrm{d}\Omega \tag{1}$$

where $I_k(\boldsymbol{r}_w, \boldsymbol{s})$ is the kth gray band radiative intensity at position $\boldsymbol{r}_w$ in the direction $\boldsymbol{s}$. $\boldsymbol{n}_w$ is the unit normal vector on the base plane, and Ω is the solid angle. This equation means that the nongray gas is replaced by a number of kth gray gases, for which gray gas intensities are calculated independently. To obtain the radiative heat flux for a two-phase mixture of nongray gases and particles, the radiative intensity at the kth gray bands at any position $\boldsymbol{r}$ along a path $\boldsymbol{s}$ through a two-phase absorbing, emitting, and scattering gas mixture with particles can be evaluated from the following radiative transfer equation (RTE) [9]:

$$\begin{aligned} &\frac{\mu}{r}\frac{\partial}{\partial r}(rI_k) - \frac{1}{r}\frac{\partial}{\partial \phi}(\eta I_k) + \xi\frac{\partial I_k}{\partial z} \\ &\quad = -(\kappa_{g,k} + \kappa_p + \sigma_{\rm sp})I_k + w_{g,k}(T_g)\kappa_{g,k}I_{b,g} \\ &\qquad + w_{p,k}(T_p)\kappa_p I_{b,p} + \frac{\sigma_{sp}}{4\pi}\int_{4\pi} I_k(s')\Phi(s', s)\, \mathrm{d}\Omega' \end{aligned} \tag{2}$$

where $\kappa_{g,k}$ is the absorption coefficient of mixture gases, κ_p and σ_{sp} are the absorption and scattering coefficients of particles, respectively. Φ is the scattering phase function that represents the effect of anisotropy; here, if the scattering is isotropic, Φ becomes unity. In addition, $w_{g,k}(T_g)$ and $w_{p,k}(T_p)$ are the weighting factors related to the kth gray band and are functions of gas temperature T_g and particle temperature T_p, respectively. As suggested and validated by Yu et al. [9], if the two-phase mixture of nongray gases and particles is not in thermal equilibrium, and if the gas and particles share all the same kth gray bands, the weighting factor for particles has the same type as that for the gas so that $w_{p,k}(T_p) = w_{g,k}(T_p)$.

To close the above two-phase RTE, the absorption coefficients and weights are calculated by using the WSGGM [10]. The particle absorption and scattering, however, are assumed gray to use the formulations obtained from Yu et al. [9] and Chui et al. [11] in the form:

$$\kappa_p = \varepsilon_p \sum_i N_i \pi r_{pi}^2 \tag{3}$$

$$\sigma_{sp} = (1 - \varepsilon_p)\sum_i N_i \pi r_{pi}^2 \tag{4}$$

where ε_p is the particle emissivity, and N_i and πr_{pi}^2 are the number density and the projected area, respectively, of the particle pertaining to group i.

Modified Discrete-Ordinates Method

The modified discrete-ordinates method (MDOM) [12] is applied to solve the RTE in Eq. (2). The advantage of the MDOM is that it is possible to choose any set of arbitrary control angles as is in the finite volume method while it still keeps a simple calculation procedure as in the conventional discrete-ordinates method (DOM). Because a more detailed discretization procedure is found in Baek and Kim [12], only the final form of the discretization equation of the MDOM using the step scheme as the spatial differencing practice is presented here as follows:

$$a_P^{mn} I_{k,P}^{mn} = a_E^{mn} I_{k,E}^{mn} + a_W^{mn} I_{k,W}^{mn} + a_T^{mn} I_{k,T}^{mn} + a_B^{mn} I_{k,B}^{mn} + S_{k,P}^{mn} \tag{5}$$

where

$$a_I^{mn} = \max(-\Delta A_i D_{ci}^{mn}, 0) \tag{6}$$

$$a_P^{mn} = \sum_{i=e,w,t,b} \max(\Delta A_i D_{ci}^{mn}, 0) + (\kappa_{g,k} + \kappa_p + \sigma_{\text{sp}})_P \Delta V \Delta\Omega^{mn} + \frac{\Delta V}{r_P}\alpha_{mn-1/2} \tag{7}$$

$$D_i^{mn} = \int_{\phi^{n-}}^{\phi^{n+}} \int_{\theta^{m-}}^{\theta^{m+}} \sin\theta(n_{i,r}\sin\theta\cos\phi + n_{i,z}\cos\theta)\,\mathrm{d}\theta\,\mathrm{d}\phi \tag{8}$$

$$S_{k,P}^{mn} = w_{g,k}(T_g)\kappa_{g,k} I_{b,g} + w_{p,k}(T_p)\kappa_p I_{b,p} + \frac{\sigma_{sp}}{4\pi}\sum_{m'=1}^{M}\sum_{n'=1}^{N} I_k^{m'n'} \Phi_{m'n'\to mn}\Delta\Omega^{m'n'} \tag{9}$$

A divergenceless flow is considered as in Carlson and Lathrop [13] to determine the $\alpha_{mn\pm 1/2}$ term due to the angular redistribution, which leads to the following recursive relation for the coefficients $\alpha_{mn\pm 1/2}$ in the form:

$$\alpha_{mn-1/2} - \alpha_{mn+1/2} = -\frac{r_P}{\Delta V}\sum_{i=e,w,t,b}\Delta A_i D_i^{mn} \tag{10}$$

where the initial condition of $\alpha_{mN+1/2} = 0$. Note that for the case of the right cylinder, the right-hand side of Eq. (10) is reduced to $-D_r^{mn}$ as shown in Baek and Kim [12].

Results and Discussion

To validate the present formulations for the analysis of axisymmetric radiative heat transfer with a two-phase mixture of nongray gases with particles, a benchmark problem in the cylindrical enclosure [9] is considered as depicted in Fig. 2. A cylindrical enclosure with 1 m in radius and 2 m in length contains a gas mixture with particles in it. The gas mixture consists of 20% H_2O, 10% CO_2, and 70% transparent gas. The particle concentration in the medium takes the value of zero (no particle), 0.01, and 0.1 kg/m^3, respectively, to simulate the effect of the amount of uniformly distributed particles on the radiative wall heat flux. The black wall temperature at the top and bottom is kept constant at 400 K while the temperature and emissivity of the side wall are 1000 K and 0.8, respectively. The scattering coefficient of the particle σ_{sp} depends on the size, number density, and emissivity of the particle as shown in Eq. (4). It is assumed that the particle scattering function is isotropic, which implies that equal amounts of energy are scattered into all directions by the particles. The particle size distribution is assumed to

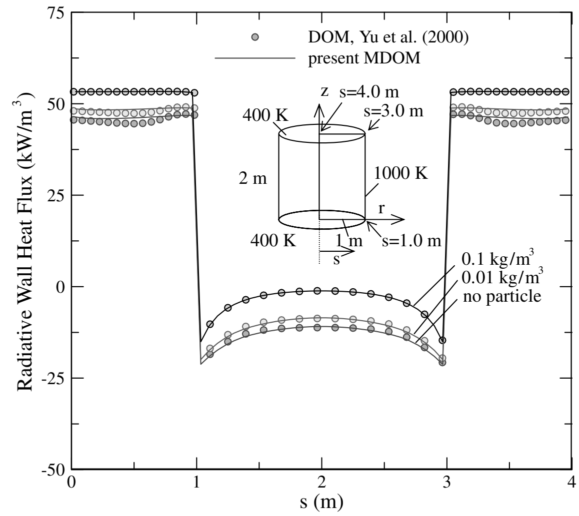

Fig. 2 Comparison of radiative wall heat flux along the walls of a cylindrical enclosure. All walls except side walls are black, while the medium of $T_g = 1000$ K comprises 20% H_2O, 10% CO_2, and 70% transparent gas with particles of $T_p = T_g$. Scattering is isotropic.

range in 50, 60, 70, 80, and 100 μm in diameter with 20% each by mass. The particle density is 1300 kg/m^3 with the particle emissivity of 0.8 [9]. Therefore, σ_{sp} takes the value of 0.037 m^{-1} in the case of $C_p = 0.01$ kg/m^3. The gas and particle temperatures are kept equal at $T_g = T_p = 1000$ K. Figure 2 shows that the present results for the effect of particle concentration on the radiative wall heat flux are in good agreement with those of Yu et al. [9].

Nominal Case

The accuracy of the radiative base heating prediction is strongly dependent on the accuracy of the plume flowfield prediction including gas and particle temperature and the optical properties used. In this work, however, the plume field is prescribed for temperature, plume gas distribution, and particle properties to facilitate the calculation of the radiative base heating. To examine the radiative heating characteristics by using the present method, results obtained by parametric study will be presented in the following by varying parameters such as scattering phase function Φ, particle concentration C_p, particle temperature T_p, and gas composition. For all the cases discussed below, isotropic scattering is assumed as a baseline so that $\Phi = 1$ is chosen, except for the case study of the effect of the scattering phase function on radiative base heating. Also, the plume cone angle δ is set to zero; therefore, only the cylindrical plume is considered here. The base plane is cold and black, and the environment outside of the plume is cold ($T_g = T_p = 0$ K) and nonparticipating ($\kappa_{g,k} = \kappa_p = \sigma_{sp} = 0$). The plume length is $Z_{\text{pl}} = 50R_{\text{ex}}$. In the case of plume emission, the temperature of the nozzle exit is set to 0 K, while the exhaust plume at $T_g = T_{\text{ref}} = 1800$ K is treated as a radiatively participating two-phase mixture of 20% H_2O, 10% CO_2, and 70% transparent gases with a cloud of particles at temperature T_p. In the case of searchlight emission, the nozzle exit temperature is maintained at $T_{\text{ref}} = 1800$ K, while all other temperatures are set to 0 K. The spatial and angular grid systems used in this work are $(N_r \times N_z) = (40 \times 70)$ and $(N_\theta \times N_\phi) = (12 \times 8)$, respectively. The various change of the grid size justified that any finer grids did not noticeably improve the accuracy of the results for all cases studied next.

Effect of Particle Concentration

The particle and gas distributions are strongly affected by the types of rocket exhaust plumes. Therefore, first the effect of particle concentration on the base heating is investigated. The results are plotted in Fig. 3 for the case of $T_g = T_p = T_{\text{ref}}$ with isotropic scattering. The particle concentrations can vary from zero to

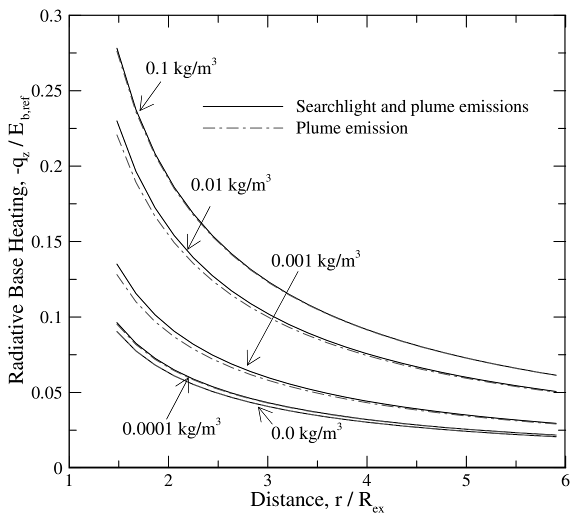

Fig. 3 The effect of particle concentration on the base heating due to searchlight emission and/or plume emission. A cylindrical isothermal exhaust plume with $Z_{pl} = 50R_{ex}$ comprises 20% H_2O, 10% CO_2, and 70% transparent gas with particles of $T_p = T_g$. Scattering is isotropic.

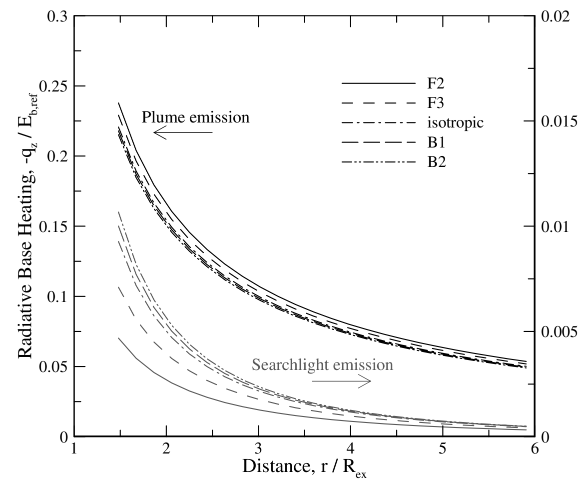

Fig. 4 The effect of the scattering phase function on the base heating due to searchlight emission or plume emission. A cylindrical isothermal exhaust plume of $Z_{pl} = 50R_{ex}$ comprises 20% H_2O, 10% CO_2, and 70% transparent gas with particles of $T_p = T_g$ with particle concentration $C_p = 0.01$ kg/m^3.

0.1 kg/m^3. Here, it is assumed that the suspending solid particle is Al_2O_3 typical in the exhaust plume of an aluminized composite propellant rocket, and the particle size distributions are 0.79, 1.28, 1.76, 2.44, and 3.95 μm with 20% each by mass [2], and the particle density is taken as 3700 kg/m^3 [5] with emissivity of 0.3 [2]. As r/R_{ex} increases, the radiative heat flux impinging on the base plane becomes smaller because the base plane views a smaller portion of the exhaust plume. In other words, the view factor toward the exhaust plume seen by a thin annulus of radius r on the base plane decreases with increasing r. When there are no particles in the plume, searchlight emission cannot be detected by the base plane because of the absence of any media by which the photons emitted from the nozzle exit are scattered into the base plane. Therefore, as shown in Fig. 3, the result from searchlight and plume emission coincides with that from plume emission only in the case of no particle used. As the particle concentration, that is, particle mass loading [5], increases from 0.0001 to 0.1 kg/m^3, the radiative heat flux on the base plane due to plume emission increases, because more particles can give more emission of radiant energy, part of which goes toward the base plane. (Note that particles do emission as well as absorption. Emission takes place as blackbody radiation while absorption does not, which leads to a positive net emission.) It must be also noted that a ratio of searchlight emission to plume emission initially increases and then decreases as the particle concentration increases. This is due to combined effects of optical thickness and scattering albedo in the plume. Such a trend characterized by an initial increase and subsequent decrease is also observed by Tan et al. [8]. The computation time required for the calculations shown in Fig. 3 is within 120 s using a 1.7-MHz personal computer.

Effect of Anisotropic Scattering

For the case of anisotropic scattering, the scattering phase function Φ is approximated by a finite series of Legendre polynomials as

$$\Phi(\mathbf{s}', \mathbf{s}) = \Phi(\cos\Psi) = \sum_{j=0}^{J} A_j P_j(\cos\Psi) \tag{11}$$

where Ψ is the scattering angle between the incoming direction $\mathbf{s}'$ and the outgoing direction $\mathbf{s}$, and A_j is the expansion coefficient. In this analysis, the forward (F2, F3) and backward (B1, B2) scattering phase functions are adopted from Kim and Lee [14]. Figure 4 shows the effect of the scattering phase function on radiative base heating due to plume or searchlight emission for the case of $T_g = T_p = T_{ref}$ and $C_p = 0.01$ kg/m^3, where plume and searchlight emission exhibit a different behavior on the radiative base heating as the scattering phase function varies, that is, plume emission is maximized for F2, which represents the strongest forward scattering, and minimized for B2, which represents the strongest backward scattering. On the other hand, searchlight emission is maximized for B2 and decreases with an enhanced forward scattering to yield the smallest emission for F2. This opposite trend is attributed to the fact that the viewing directions of the nozzle exit and the plume toward the base plane are contrary to each other. Figure 4 also indicates that searchlight emission is very small compared to plume emission because the amount of photons coming out of the nozzle is much smaller than that coming out of a relatively large volume of the plume. This searchlight emission is less than about 5% of plume emission for all cases.

Effect of Particle Temperature

It is very likely to observe in an actual rocket plume that the gas temperature may be somewhat different from the particle temperature when they escape the nozzle exit too rapidly to reach a thermal equilibrium condition with each other. In general, particle temperatures on the nozzle and plume are always higher than that of the surrounding gas because they cool more slowly than the gas during flow expansion in the nozzle. Figure 5 shows the effect of particle temperature on radiative heating in the case of isotropic scattering with $C_p = 0.01$ kg/m^3 for three different particle temperature distributions. As the particle temperature increases to 200 and 400 K higher than that of the gas, radiative base heating increases because more radiant energy is emitted and scattered from the particle. Searchlight emission, however, remains almost constant regardless of the particle temperature variations.

Effect of Gas Composition

Finally, the effect of the nongray gas composition on the radiative base heating is explored with three different cases, that is, a single gaseous species, a mixtures of gases, and mixtures of gases with particles, as shown in Fig. 6. When there is no particle in the plume, only plume emission is considered because if there is no scattering, no searchlight emission occurs. Here, it can be found that doubling the gaseous species concentration increases the radiative base heating regardless of the existence of the particles. This result is caused by the fact that the more the participant gases exist, the more emission and scattering of the radiative thermal energy take place toward the base plane. However, although doubling the H_2O and CO_2 concentration increases the radiative heat flux to the base plane, as shown in Fig. 6, this increase is quite small if the particles exist.

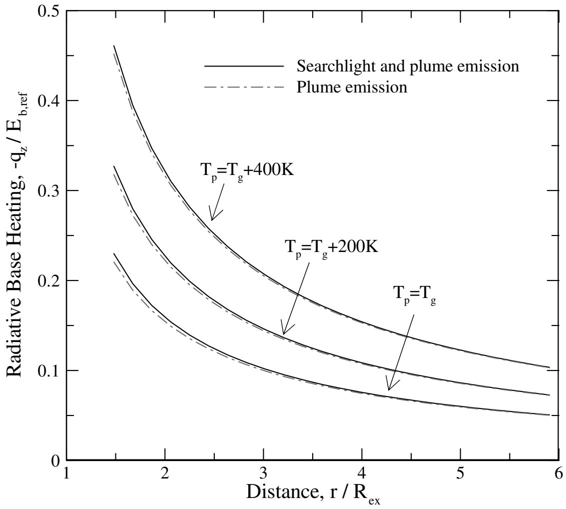

Fig. 5 The effect of particle temperature on the base heating due to searchlight emission and/or plume emission. A cylindrical isothermal exhaust plume of $Z_{pl} = 50R_{ex}$ comprises 20% H_2O, 10% CO_2, and 70% transparent gas with particles concentration of $C_p = 0.01\ kg/m^3$. Scattering is isotropic.

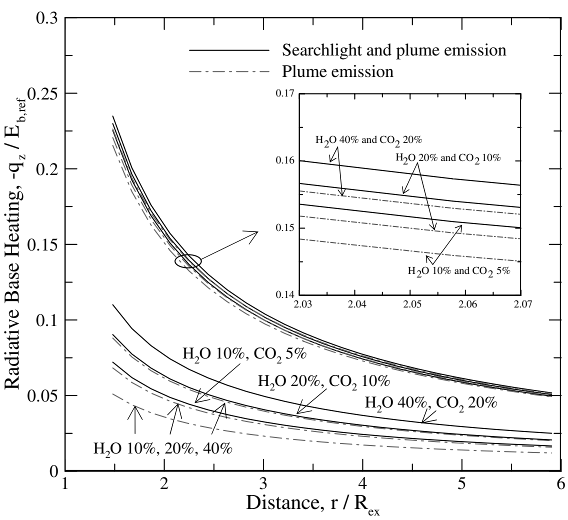

Fig. 6 The effect of gas composition on the base heating due to searchlight emission and/or plume emission. $Z_{pl} = 50R_{ex}$, $C_p = 0.01\ kg/m^3$, and $T_p = T_g$. Scattering is isotropic.

Conclusions

The modified discrete-ordinates method with a two-phase mixture of nongray gases and a cloud of particles has been effectively applied to the radiative base heating due to plume and searchlight emission. After validating the present formulations in the axisymmetric radiation case with a two-phase mixture of gases and particles, numerical investigations are made on the evaluation of the radiative base heating due to plume and searchlight emission to examine the effects of particle concentration, scattering phase function, particle temperature, and nongray gas composition on the radiative base heating in an isothermal cylindrical plume with nongray gas and Al_2O_3 particles. All the cases in this study using the simple plume models show that the base plane is predominantly heated by plume emission rather than searchlight emission. The radiative base heating increases as the particle concentration and/or temperature increase. The forward scattering of particles increases plume emission while it decreases searchlight emission. The gas composition, however, does not significantly influence the radiative base heating.

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